

# 7.5

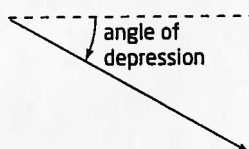
## Solve Problems Involving Right Triangles

The primary trigonometric ratios are applied in many areas of study, including architecture, engineering, astronomy, medicine, and criminal detection. For example, have you ever seen a television show or movie in which forensic evidence is used to recreate the events of an accident or crime? How can scientists use a few tiny clues, such as bloodstains, to piece together what actually happened? How could trigonometry be useful?



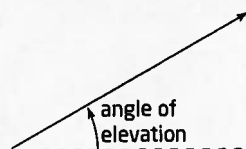
### angle of depression

- angle measured below the horizontal
- also called the **angle of declination**



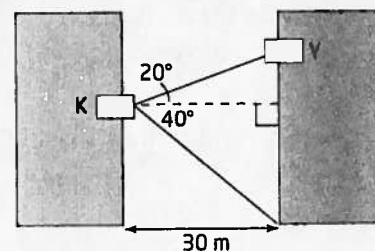
### angle of elevation

- angle measured above the horizontal
- also called the **angle of inclination**



### Example 1 Angles of Depression and Elevation

Kim and Yuri live in apartment buildings that are 30 m apart, as shown. The **angle of depression** from Kim's balcony to where Yuri's building meets the ground is  $40^\circ$ . The **angle of elevation** from Kim's balcony to Yuri's balcony is  $20^\circ$ .



- How high is Kim's balcony above the ground, to the nearest metre?
- How high is Yuri's balcony above the ground, to the nearest metre?

### Solution

- Simplify the problem by focusing on the lower right triangle.

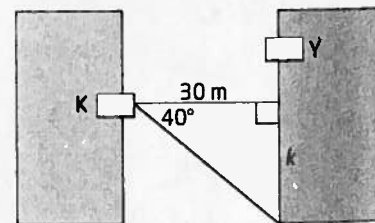
$$\tan 40^\circ = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 40^\circ = \frac{k}{30}$$

$$30 \tan 40^\circ = k \quad \text{Multiply both sides by 30.}$$

$$25.17 \doteq k$$

Kim's balcony is about 25 m above the ground.



b) Focus on the upper right triangle.

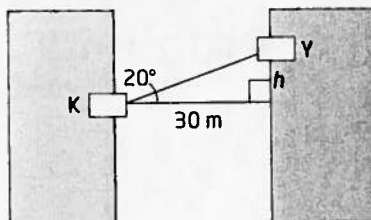
$$\tan 20^\circ = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 20^\circ = \frac{h}{30}$$

$$30 \tan 20^\circ = h$$

$$10.92 \doteq h$$

Multiply both sides by 30.



Yuri's balcony is about 11 m higher than Kim's. Add this to the height of Kim's balcony to find the height of Yuri's balcony,  $y$ .

$$y = h + k$$

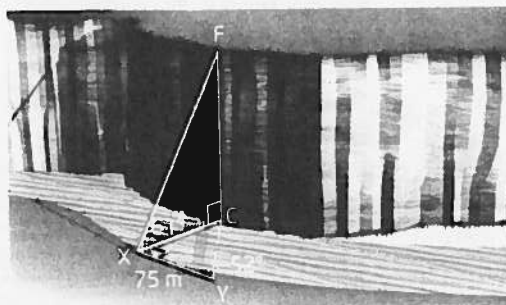
$$= 25 + 11$$

$$= 36$$

Yuri's balcony is about 36 m above the ground.

## Example 2 Solve a Three-Dimensional Problem

A theodolite is an instrument used by a surveyor to measure horizontal and vertical angles. Measurements are taken in order to find the height of a cliff on the other side of a river, as shown.



Find the height of the cliff, to the nearest metre.

### Solution

Use  $\triangle CYX$  to find the width of the river,  $CX$ . Then, use  $\triangle CFX$  to find the height of the cliff,  $CF$ . Focus on  $\triangle CYX$  first.

Apply the tangent ratio to find  $CX$ .

$$\tan \angle Y = \frac{CX}{XY}$$

$$\tan 52^\circ = \frac{CX}{75}$$

$$75 \tan 52^\circ = CX$$

$$95.996 \doteq CX$$

Multiply both sides by 75.

$CX$  is shared by both triangles. Use it to find the height of the cliff.



### Literary Connections

In this situation, using a single letter to identify a side can lead to confusion. For example, side  $c$  could refer to the hypotenuse in  $\triangle CFX$  or the 75-m side of  $\triangle CYX$ . Use endpoints to distinguish line segments,  $FX$  and  $XY$  in this case, to avoid confusion.